

Confinement in the presence of external fields and axions

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Abstract

For a theory with a pseudo scalar coupling $\phi F\tilde{F}$ and in the case that there is a constant electric or magnetic strength expectation value, we compute the interaction potential within the structure of the gauge-invariant but path-dependent variables formalism. While in the case of a constant electric field strength expectation value the static potential remains Coulombic, in the case of a constant magnetic field strength the potential energy is the sum of a Yukawa and a linear potentials, leading to the confinement of static charges.

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I. INTRODUCTION

One of the key issues facing *QCD* is understanding confinement of quarks and gluons. In fact, a linearly increasing quark-antiquark pair static potential provides the simplest criterion for confinement, although unfortunately there is up to now no known way to analytically derive the confining potential from first principles. However, as is well known, it has been

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approached from many different techniques and ideas, like lattice gauge theories [1], non perturbative solutions of Schwinger-Dyson's equations [2]. Other authors also associate confinement with the existence of a non trivial vacuum structure, where the chromomagnetic field strength acquires a non zero expectation value [3].

In fact non vanishing expectation values can have dramatic consequences in everything that concerns the infrared properties of a theory. An interesting model where these effects have been studied to a certain extent is the "axion-gauge field" system, where a scalar field ϕ (the "axion") is coupled to gauge fields via the interaction term

$$\mathcal{L}_I = \frac{g}{8} \phi \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}. \quad (1)$$

This theory experiences mass generation when ϕ develops a space dependent expectation value [4] and tachyonic mass generation when a time dependent expectation value appears [5]. Mass generation is also achieved when the gauge field $F_{\mu\nu}$ takes a magnetic type expectation value [6]. If $F_{\mu\nu}$ takes an electric type expectation value, tachyonic mass generation takes place [6]. Thus, in order to gain further insight into the physics presented by this theory, in this paper we will focus attention on the static potential between charged fields. The purpose here is to investigate the effects of the external expectation value field strength on the interaction energy.

The interaction energy between static charges is a tool of considerable interest which is expected to provide the foundation for understanding confinement, and its physical content can be understood when a correct separation of the physical degrees of freedom is made. Previously, we proposed a general framework for studying the confining and screening nature of the static potential in gauge theories in terms of the gauge-invariant but path-dependent field variables [7]. An important feature of this methodology is that it provides a physically-based alternative to the usual Wilson loop approximation. When we compute in this way the static potential for the model described in [6], which contains the term (1), in the presence of an external field strength which can be either electric or magnetic, the result of this calculation is rather unexpected in the magnetic case: It is shown that the interaction

energy is the superposition of a Yukawa and a linear potentials, that is, the confinement between static charges is obtained. On the other hand, in the case of a constant electric field strength expectation value the static potential remains Coulombic, that is, the interaction energy does not exhibit any sensitive modification. Actually, the linear confining potential seems to be associated only with the magnetic field strength expectation value.

II. INTERACTION ENERGY

Before going to the derivation of the interaction energy, we will describe very briefly the model under consideration. We start from the following effective Lagrangian [6]:

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{g^2}{16}\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle\varepsilon^{\rho\sigma\gamma\delta}\langle F_{\rho\sigma}\rangle f_{\alpha\beta}\frac{1}{\square + m_A^2}f_{\gamma\delta}. \quad (2)$$

where $\langle F_{\mu\nu}\rangle$ represents the constant classical background (which is a solution of the classical equations of motion), and m_A is the mass for the axion field. Here, $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ describes a small fluctuation around the background. We also mention that the above Lagrangian arose after using $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle\langle F_{\alpha\beta}\rangle = 0$ (which holds for a pure electric or a pure magnetic background), and integrating out the axion fields ϕ .

By introducing $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle \equiv v^{\alpha\beta}$ and $\varepsilon^{\rho\sigma\gamma\delta}\langle F_{\rho\sigma}\rangle \equiv v^{\gamma\delta}$, it follows that the expression (2) can be rewritten as

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{g^2}{16}v^{\alpha\beta}f_{\alpha\beta}\frac{1}{\square + m_A^2}v^{\gamma\delta}f_{\gamma\delta}, \quad (3)$$

still, the tensor $v^{\alpha\beta}$ is not arbitrary, but must satisfy $\varepsilon^{\mu\nu\alpha\beta}v_{\mu\nu}v_{\alpha\beta} = 0$.

A. Magnetic case

As stated, our main objective is to calculate the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows), following the conventional path via the expectation value of the Hamiltonian in the physical state $|\Phi\rangle$, which we will denote by $\langle H \rangle_\Phi$. The Lagrangian (3) then becomes

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{g^2}{16}v^{0i}f_{0i}\frac{1}{\square + m_A^2}v^{0k}f_{0k} - A_0J^0, \quad (4)$$

where J^0 is an external current, $(\mu, \nu = 0, 1, 2, 3)$ and $(i, k = 1, 2, 3)$.

We now proceed to obtain the Hamiltonian. For this we consider the Hamiltonian formulation of this theory. The canonical momenta obtained from (4) are

$$\Pi^0 = 0, \quad (5)$$

and

$$\Pi_i = D_{ij}E_j, \quad (6)$$

where $E_i \equiv F_{i0}$ and $D_{ij} \equiv \left(\delta_{ij} - \frac{g^2}{8}v_{i0}\frac{1}{\square + m_A^2}v_{j0}\right)$. Since D is a nonsingular matrix ($\det D = 1 - \frac{g^2}{8}\frac{\mathbf{v}^2}{\square + m_A^2} \neq 0$) with $\mathbf{v}^2 \equiv v^{i0}v^{i0}$, there exists the inverse of D and from Eq.(6) we obtain

$$E_i = \frac{1}{\det D} \left\{ \delta_{ij} \det D + \frac{g^2}{8}v_i\frac{1}{\square + m_A^2}v_j \right\} \Pi_j. \quad (7)$$

The canonical Hamiltonian corresponding to (4) is

$$H_C = \int d^3x \left\{ -A_0 (\partial_i \Pi^i - J^0) + \frac{1}{2} \mathbf{\Pi}^2 + \frac{g^2}{16} \frac{(\mathbf{v} \cdot \mathbf{\Pi})^2}{(\square + M^2)} + \frac{1}{2} \mathbf{B}^2 \right\}, \quad (8)$$

where $M^2 \equiv m_A^2 - \frac{g^2}{8}\mathbf{v}^2$ and \mathbf{B} is the magnetic field. Demanding that the primary constraint $\Pi_0 = 0$ be preserved in the course of time, one obtains the secondary Gauss law constraint of the theory as $\Gamma_1(x) \equiv \partial_i \Pi^i - J^0 = 0$. The preservation of Γ_1 for all times does not give rise to any further constraints. The theory is thus seen to possess only two constraints, which are first class, therefore the theory described by (4) is a gauge-invariant one. The extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x (c_0(x) \Pi_0(x) + c_1(x) \Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are the Lagrange multiplier fields. Moreover, it is straightforward to see that $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither A^0 nor Π^0 are of interest in describing the system and may be discarded from the theory. Thus the Hamiltonian takes the form

$$H = \int d^3x \left\{ \frac{1}{2} \mathbf{\Pi}^2 + \frac{g^2}{16} \frac{(\mathbf{v} \cdot \mathbf{\Pi})^2}{(\square + M^2)} + \frac{1}{2} \mathbf{B}^2 + c(x) (\partial_i \Pi^i - J^0) \right\}, \quad (9)$$

where $c(x) = c_1(x) - A_0(x)$.

To quantize the theory using Dirac's procedure [8] we introduce a supplementary condition on the vector potential such that the full set of constraints becomes second class. For this purpose, we could choose, for example, the gauge-fixing condition [7]

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0, \quad (10)$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x - \xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. In this case, the only nonvanishing equal-time Dirac bracket is

$$\{A_i(x), \Pi^j(y)\}^* = \delta_i^j \delta^{(3)}(x - y) - \partial_i^x \int_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \quad (11)$$

In passing we recall that the transition to quantum theory is made by the replacement of the Dirac brackets by the operator commutation relations according to

$$\{A, B\}^* \rightarrow (-i)[A, B]. \quad (12)$$

We are now in a position to evaluate the interaction energy between pointlike sources in the model under consideration, where a fermion is localized at \mathbf{y}' and an antifermion at \mathbf{y} . From our above discussion, we see that $\langle H \rangle_\Phi$ reads

$$\langle H \rangle_\Phi = \langle \Phi | \int d^3x \left\{ \frac{1}{2} \mathbf{\Pi}^2 + \frac{g^2}{16} \frac{(\mathbf{v} \cdot \mathbf{\Pi})^2}{(\square + M^2)} + \frac{1}{2} \mathbf{B}^2 \right\} | \Phi \rangle. \quad (13)$$

Next, as was first established by Dirac [9], the physical state can be written as

$$| \Phi \rangle \equiv | \overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \rangle = \overline{\psi}(\mathbf{y}) \exp \left(ie \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i(z) \right) \psi(\mathbf{y}') | 0 \rangle, \quad (14)$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a spacelike path starting at \mathbf{y}' and ending at \mathbf{y} , on a fixed time slice. From this we see that the fermion fields are now dressed by a cloud of gauge fields. As mentioned before,

the fermions are taken to be infinitely massive (static). Consequently, we can write Eq.(13) as

$$\langle H \rangle_{\Phi} = \langle \Phi | \int d^3x \left\{ \frac{1}{2} \mathbf{\Pi}^2 - \frac{g^2 (\mathbf{v} \cdot \mathbf{\Pi})^2}{16 \nabla^2 - M^2} \right\} | \Phi \rangle, \quad (15)$$

with $\partial_i \partial^i = -\nabla^2$.

From the foregoing Hamiltonian discussion, we first note that

$$\Pi_i(\mathbf{x}) \left| \overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \right\rangle = \overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \Pi_i(\mathbf{x}) |0\rangle + e \int_{\mathbf{y}}^{\mathbf{y}'} d\mathbf{z}_i \delta^{(3)}(\mathbf{z} - \mathbf{x}) | \Phi \rangle. \quad (16)$$

Combining Eqs.(15) and (16), we have

$$\langle H \rangle_{\Phi} = \langle H \rangle_0 + V_1 + V_2, \quad (17)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$.

The V_1 term is given by

$$V_1 = \frac{e^2}{2} \int_{\mathbf{y}}^{\mathbf{y}'} dz'_i \partial_i^{z'} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \partial_z^i G(\mathbf{z}', \mathbf{z}), \quad (18)$$

where G is the Green function

$$G(\mathbf{z}', \mathbf{z}) = \frac{1}{4\pi} \frac{e^{-M|\mathbf{z}' - \mathbf{z}|}}{|\mathbf{z}' - \mathbf{z}|}. \quad (19)$$

By means of Eq.(19) and remembering that the integrals over z^i and z'_i are zero except on the contour of integration, the term (18) reduces to the Yukawa-type potential after subtracting the self-energy terms, that is,

$$V_1 = -\frac{e^2}{4\pi} \frac{e^{-M|\mathbf{y} - \mathbf{y}'|}}{|\mathbf{y} - \mathbf{y}'|}. \quad (20)$$

We now come to the V_2 term, which is given by

$$V_2 = \frac{e^2 m_A^2}{2} \int_{\mathbf{y}}^{\mathbf{y}'} dz'^i \int_{\mathbf{y}}^{\mathbf{y}'} dz^i G(\mathbf{z}', \mathbf{z}). \quad (21)$$

In order to compute V_2 , we make use of the Green function (19) in momentum space

$$\frac{1}{4\pi} \frac{e^{-M|\mathbf{z}' - \mathbf{z}|}}{|\mathbf{z}' - \mathbf{z}|} = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot (\mathbf{z}' - \mathbf{z})}}{\mathbf{k}^2 + M^2}. \quad (22)$$

Thus, by employing relation (22) we can reduce Eq.(21) to

$$V_2 = e^2 m_A^2 \int \frac{d^3 k}{(2\pi)^3} [1 - \cos(\mathbf{k} \cdot \mathbf{r})] \frac{1}{(\mathbf{k}^2 + M^2)} \frac{1}{(\hat{\mathbf{n}} \cdot \mathbf{k})^2}, \quad (23)$$

where $\hat{\mathbf{n}} \equiv \frac{\mathbf{y}-\mathbf{y}'}{|\mathbf{y}-\mathbf{y}'|}$ is a unit vector and $\mathbf{r} = \mathbf{y} - \mathbf{y}'$ is the relative vector between the quark and antiquark. Since $\hat{\mathbf{n}}$ and \mathbf{r} are parallel, we get accordingly

$$V_2 = \frac{e^2 m_A^2}{8\pi^3} \int_{-\infty}^{\infty} \frac{dk_r}{k_r^2} [1 - \cos(k_r r)] \int_0^{\infty} d^2 k_T \frac{1}{(k_r^2 + k_T^2 + M^2)}, \quad (24)$$

where k_T denotes the momentum component perpendicular to \mathbf{r} . We may further simplify Eq.(24) by doing the k_T integral, which leads immediately to the result

$$V_2 = \frac{e^2 m_A^2}{8\pi^2} \int_{-\infty}^{\infty} \frac{dk_r}{k_r^2} [1 - \cos(k_r r)] \ln \left(1 + \frac{\Lambda^2}{k_r^2 + M^2} \right), \quad (25)$$

where Λ is an ultraviolet cutoff. We also observe at this stage that similar integral was obtained independently in Ref. [10] in the context of the dual Ginzburg-Landau theory by an entirely different approach.

Now, we move on to compute the integral (25). To this end it is advantageous to introduce a new auxiliary parameter ε by making in the denominator of the integral (25) the substitution $k_r^2 \rightarrow k_r^2 + \varepsilon^2$. This allows us to obtain a form more comfortable to handle the integral. Hence we evaluate $\lim_{\varepsilon \rightarrow 0} \tilde{V}_2$, that is,

$$V_2 \equiv \lim_{\varepsilon \rightarrow 0} \tilde{V}_2 = \lim_{\varepsilon \rightarrow 0} \frac{e^2 m_A^2}{8\pi^2} \int_{-\infty}^{\infty} \frac{dk_r}{(k_r^2 + \varepsilon^2)} [1 - \cos(k_r r)] \ln \left(1 + \frac{\Lambda^2}{k_r^2 + M^2} \right). \quad (26)$$

The integration on the k_r -complex plane yields

$$\tilde{V}_2 = \frac{e^2 m_A^2}{8\pi} \left(\frac{1 - e^{-\varepsilon|\mathbf{y}-\mathbf{y}'|}}{\varepsilon} \right) \ln \left(1 + \frac{\Lambda^2}{M^2 - \varepsilon^2} \right). \quad (27)$$

Taking the limit $\varepsilon \rightarrow 0$, expression (27) then becomes

$$V_2 = \frac{e^2 m_A^2}{8\pi} |\mathbf{y} - \mathbf{y}'| \ln \left(1 + \frac{\Lambda^2}{M^2} \right). \quad (28)$$

This, together with Eq.(20), yields finally

$$V(L) = -\frac{e^2}{4\pi} \frac{e^{-ML}}{L} + \frac{e^2 m_A^2}{8\pi} L \ln \left(1 + \frac{\Lambda^2}{M^2} \right), \quad (29)$$

where $L \equiv |\mathbf{y} - \mathbf{y}'|$.

It is worth noting here that this is exactly the result obtained in Ref. [10] in the context of the dual Landau-Ginzburg theory. But we do not think that the agreement is an accidental coincidence. Also, the massive Abelian antisymmetric tensor gauge theory displays the same behavior [11,13]. In other words, there is a class of models which can predict this interaction energy.

B. Electric case

We now want to extend what we have done to the case $v^{0i} = 0$ and $v^{ij} \neq 0$ (referred to as the electric one in what follows). In such a case the Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{g^2}{16}v^{ij}f_{ij}\frac{1}{\square + m_A^2}v^{kl}f_{kl} - A_0J^0, \quad (30)$$

($\mu, \nu = 0, 1, 2, 3$) and ($i, j, k, l = 1, 2, 3$).

The above Lagrangian will be the starting point of the Dirac constrained analysis. The canonical momenta following from Eq.(30) are $\Pi^\mu = f^{\mu 0}$, which results in the usual primary constraint $\Pi^0 = 0$ and $\Pi^i = f^{i0}$. Defining the electric and magnetic fields by $E^i = F^{i0}$ and $B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$, respectively, the canonical Hamiltonian assumes the form

$$H_C = \int d^3x \left\{ \frac{1}{2}\mathbf{E}^2 + \frac{1}{2}\mathbf{B}^2 + \frac{g^2}{16}\epsilon_{ijm}\epsilon_{klm}v^{ij}B^m\frac{1}{\square + m_A^2}v^{kl}B^n - A_0(\partial_i\Pi^i - J^0) \right\}. \quad (31)$$

Time conservation of the primary constraint leads to the secondary constraint $\Gamma_1(x) \equiv \partial_i\Pi^i - J^0 = 0$, and the time stability of the secondary constraint does not induce more constraints, which are first class. It should be noted that the constrained structure for the gauge field is identical to the usual Maxwell theory. Notwithstanding, in order to put our discussion into context it is useful to summarize the relevant aspects of the analysis described previously [12]. In view of this situation, we pass now to the calculation of the interaction energy.

Following our earlier procedure, we will compute the expectation value of the Hamiltonian in the physical state $|\Phi\rangle$ (Eq. (14)). That is,

$$\langle H \rangle_\Phi = \langle \Phi | \int d^3x \left\{ \frac{1}{2} \mathbf{E}^2 \right\} | \Phi \rangle. \quad (32)$$

Taking into account the above Hamiltonian structure, the interaction takes the form

$$\langle H \rangle_\Phi = \langle H \rangle_0 + \frac{e^2}{2} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \int_{\mathbf{y}}^{\mathbf{y}'} dz'_i \delta^{(3)}(\mathbf{z} - \mathbf{z}'), \quad (33)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. Once again, the integrals over z_i and z'_i are zero except on the contour of integrations, accordingly one obtains the following interaction energy:

$$V = \frac{e^2}{2} k |\mathbf{y} - \mathbf{y}'|, \quad (34)$$

where $k = \delta^{(2)}(0)$. This expression shows that special care has to be exercised in order to clarify the appearance of this peculiar result, as was discussed elaborately in [12]. It may be recalled, however, that the origin of the divergence is quite clear, so that it is possible to extract the Coulomb potential from the infinite contribution. Notice that the origin of the divergent factor k is due to the fact that the thickness of the string is nonvanishing only on the contour of integration. We recall that a suitable examination of the term $\frac{e^2}{2} \int d^3x \left(\int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{x} - \mathbf{z}) \right)^2$ reproduces exactly the expected Coulomb interaction between charges after subtracting the self-energy term [12], hence Eq.(34) reduces to

$$V(L) = -\frac{1}{4\pi} \frac{1}{L}, \quad (35)$$

where $L \equiv |\mathbf{y} - \mathbf{y}'|$.

III. FINAL REMARKS

We briefly summarize the results obtained so far. By using the gauge-invariant but path-dependent formalism, we have studied the static potential for the system consisting of a gauge field interacting with a massive axion field in the case when there are nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$. The constant gauge field configuration is a solution of the classical equations of motion.

While in the case when $\langle F_{\mu\nu} \rangle$ is electric-like no unexpected features are found, we find that the case when $\langle F_{\mu\nu} \rangle$ is magnetic-like is totally different. In fact, when $\langle F_{\mu\nu} \rangle$ is magnetic-like, the potential between static charges displays a Yukawa piece plus a linear confining piece. Unexpectedly, a confining potential between static charges appears in this case. It is interesting to note that the requirement that $\langle F_{\mu\nu} \rangle$ be magnetic in order to get confining behavior coincides with ideas concerning the nature of the QCD vacuum, where a nontrivial magnetic field strength must be present in the vacuum [3].

In the non-Abelian generalization of this model similar effects should appear.

Further investigations of the relations between our work and the magnetic models of the QCD vacuum in Ref. [3] have to be performed. Also one should address the question of what is the physical origin of the axion field, it is probably some kind of a bound state, if our theory represents an effective approach to QCD .

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